THURSDAY, FEBRUARY 22, 1894.

BOLTZMANN ON MAXWELL.

Lectures on Maxwell's Theory of Electricity and Light. Part ii. By Dr. Ludwig Boltzmann. 8vo. pp. 166. (Leipzig: Barth, 1893.)

THIS second part of Dr. Boltzmann's account of Maxwell's electromagnetic theory is written from a somewhat different point of view from the first part. The first part presents the theory from the mathematical point of view of a dynamical system whose generalised coordinates are known. This one presents the theory from the physical point of view of a continuous medium whose intimate structure is indeed not fully understood, but whose changes of structure can be fully represented by certain vectors. Although Maxwell has presented the subject from both points of view, the one which really determines the form of his work, and that appears to have led him in his investigations, is the physical view of this second part. A purely analytical view is hardly ever as suggestive as a physical and geometrical one. This latter one suggests extensions, suggests advances in a way that purely analytical investigations seldom do. Compare, for instance, Ampère's investigation of the action of elements of currents on one another with Faraday's treatment of the same subject. The latter has suggested the whole of the recent advances, the working of the ether, the identity of light and electromagnetic waves. The former was magnificent, brilliant no doubt, but it was cold and dead.

In the preface to this second part, Dr. Boltzmann explains the absence of diagrams, marginal notes, &c. which appeared in the former part. He says a friend conveyed to him the valuable criticism that "Your book is dear." He has consequently left out the embellishments that Englishmen love, as being too expensive for the poor German student, and has only left the motto, which costs nothing, to wit :—

> "War es ein Gott, der diese Zeichen schrieb
> Die mit geheimnissvoll verborg'nem Trieb
> Die Kräfte der Natur um mich enthüllen
> Und mir das Herz mit stiller Freude füllen."

And even this he has not taken from that classic lore that Englishmen delight in, but from a German poet.

Although he regrets the dark and inconsequent character of much of Maxwell's work, he congratulates scientific men that this has left them the more to do. For himself, he only claims to be an exponent of Maxwell's views, and hopes that he may succeed in helping students to understand them.

Electromagnetic equations lend themselves to a great variety of interpretations by analogy with displacements of a medium. They are a system of vectors related to one another by a very simple method of derivation each from the last, by the process of what Dr. Boltzmann says the English call "curling." Starting with the vector potential, the magnetic force is its curl, and the curl of the magnetic vector is the time rate of variation of the electric vector. Any one of this system of vectors may be likened to the displacement or velocity of an incompressible medium, and hence we have one system in which the vector potential is so likened, one in which the electric displacement is so likened, and one in which the magnetic displacement is so likened. These latter two analogies have been the favourite ones. Maxwell frequently speaks in terms of the system in which the electric vector is likened to a displacement of an incompressible medium, and the likening of the magnetic vector to a flow is quite common. In these systems electric energy is generally considered as potential, and magnetic energy as kinetic. Dr. Boltzmann, however, likens the vector potential, which he calls the tonus, and which Mr. Oliver Heaviside relegates to the realms of merely convenient suppositions, to a displacement of the medium; and in accordance with this analogy the magnetic energy becomes potential, and the electric vector, which is proportional to the rate of variation of the vector potential, being thus a velocity, requires the assumption that electric energy is kinetic. An obvious difficulty arises here from the necessity of making an electric current an acceleration which cannot of course be constant for ever. In this connection it may be worth while observing that the possible existence of one closed surface inside another with static lines of the electric vector between them makes it necessary to assume either (1) that the vector potential represents a twist round its line, and not a displacement along it; or (2) that it is a displacement up some lines and down others; or (3) that there are sources and sinks of the ether where there is electrification, because without sources and sinks we cannot have a continuous flow going on out from a closed inner surface to a closed outer one. If the tonic vector be a twist, the magnetic vector will be of the nature of a Δ^2, and it would be this structure which should be elastically resisted, and not a twist, as Dr. Boltzmann's and Mr. Larmor's assumptions give. This would return somewhat to Mr. Glazebrook's proposal of years ago. Of course, a complex change of structure, such as a combination of 1 and 2, or any other change, such as crystallisation in a hemihedral crystal, would be a possible solution. In a hemihedral crystalline form, because its two ends must differ in sign. Gravity is probably due to a change of structure produced by the presence of matter, which is analogous to a non-hemihedral crystallisation because it is always attractive; there seems no reason to suppose that any bodies exist possessing negative gravitation, the supposed levity of the old philosophers.

To the vector potential, Boltzmann gives Faraday's name of electrotonic state at the point, or, shortly, the tonus of the element of volume. The rate of change of this tonus is the electric vector E, and the kinetic energy due to it is the electric energy per unit vol.

$$T = K/8\pi \,.\, E^2$$

This tonic strain is accompanied in general by a tonic stress depending on the curl of the tonic vector which is the magnetic vector, H = curl E, and a corresponding potential energy

$$V = \frac{\nu}{2} H^2.$$

All this is most interesting in connection with Mr. Larmor's recent papers. He uses Maxwell's analysis in which the magnetic energy is kinetic, and consequently assumes the magnetic vector to be a flow, which he has

©1894 Nature Publishing Group

pointed out can exist as an irrotational one without reaction in MacCullagh's medium. This same observation of course applies to Dr. Boltzmann's analysis, the difference being that vortex rings would be rings of magnetic current instead of electric current, and atoms would act like electric diads instead of elementary magnets. The existence of unclosed lines of electric vector, however, seems to make this simple interpretation of Dr. Boltzmann's analysis impossible, and as we cannot in general substitute whirl for flow in fluid motion, the vortex ring analysis could not be applied if Dr. Boltzmann's electric vector were interpreted as a whirl; and hence Mr. Larmor's investigation seems confined to the interpretation he has given.

Having assumed that the medium is such as to react elastically against curl of the tonus (τ), and his fundamental equations thus being—

$$(1),\ E=\dot{\tau}\ ;\ (3),\ T=K/8\pi\ .\ E^2$$
$$(2),\ H=\text{curl}\ \tau\ ;\ (4),\ V=\nu/2\ .\ H^2$$

he proceeds to deduce the equation corresponding to (2), namely, (5)$K\dot{E}=\text{curl}\ H$ by applying Hamilton's principle to $T-V$ in a way which is well known. He now remarks that in accordance with his dynamical principles $K\dot{E}$ is rate of change of momentum, and he adds to curl H any external impressed forces, which he divides into two classes: (1) those due to reversible causes, such as electromotive of contact, chemical action, &c. (F); and (2) those due to irreversible causes, such as ohmic resistance, &c., which are proportional to the electric vector, and thus obtains this equation in the form

$$KE=\text{curl}\ H-4\pi C\ (E+F)$$

It would thus seem as if the electric conduction current were a different thing from a changing electric displacement, though both depending on curl H. This arises from the difficulty noticed above, and seems to require careful consideration. By judicious theories as to the function of the matter in stopping the continual acceleration without uncurling the H, the difficulty can be surmounted. In order to get over all the difficulties of discontinuities at the surfaces of bodies, Dr. Boltzmann assumes that the properties of the ether vary rapidly but continuously in passing across a surface, so that he can assume that these equations apply everywhere.

Depending on his dynamical basis, Dr. Boltzmann has obtained the following dimensions for electromagnetic quantities, which, of course, differ entirely from both the electric and magnetic systems of units—

$$[E]=[LT^{-1}],\ [K]=[ML^{-3}]$$
$$[H]=[ML^{-1}T^{-2}]\text{*}\quad[\mu]=[M^{-1}LT^{-2}]$$

After a short discussion as to the possibility of founding the science upon a purely analytical basis, by assuming equations and showing that they lead to true results, which is the basis of Hertz's method, and a short criticism of this method as applied by Hertz, Dr. Boltzmann proceeds to show how the old equations of action at a distance and von Helmholtz's work are connected with Maxwell's view of the subject. His treatment of superficial effects by means of a rapid variation of structure of the ether at the surface of solids, seems essentially the same as Mr. O. Heaviside has advocated in opposition to von Helmholtz's double electric layers. All this analytical method is, of course, necessary and interesting in what may be called a transition work, one that concerns those who have been brought up under one school of thought and are entering another; a sort of epistle to the Hebrews, a college between youth and manhood. It concerns the past rather than the future, towards which we should press, forgetting those things that are behind.

Although Dr. Boltzmann has left out all embellishments, he has had pity on his readers. There are necessarily included in a transition work of this kind innumerable formulæ, of which 168 are frequently referred to, and these he has collected into two folding sheets, each of five folds, at the end of his work. This is most considerate. Books dealing with many formulæ might well follow suit, although it certainly is a little terrifying to have 168 formulæ presented as the outcome of the book in a way that necessarily attracts the attention of anyone who thinks of reading it. Those who are frightened by this should, however, recollect that the whole subject of electromagnetism depends on only four very simple equations. Dr. Boltzmann would have much simplified his work if he had adopted any vector symbolism.

It is to be hoped that this part of Dr. Boltzmann's work, as well as the former part, will soon be translated, and so made easily accessible to English students. The work of a great master, the product of a great mind, helps all men who can understand it.

THE STORY OF THE SUN.

The Story of the Sun. By Sir Robert Ball, LL.D. (London: Cassell and Co., 1893.

THERE is no more interesting chapter in science than that which deals with our great central luminary. Its story has been gradually gaining in interest since the first application of the telescope to its study by Galileo, and since the advent of the spectroscope our knowledge of solar phenomena has advanced by leaps and bounds. At the present time the scrutiny of the sun is more minute and continuous than ever, and the constant acquisition of fresh information sufficiently explains the need for additional works on the subject, or for new editions of old ones.

The author of the book before us does not approach the subject as a practical investigator in this branch of astronomy, and his efforts are therefore chiefly intended for the delectation of that class of readers for which he chiefly caters. The first thing that strikes one on glancing through the pages of the book is the great variety of the matter which it contains, and one begins to wonder if he has mistaken the title of the volume. It is not too much to say that nearly every department of astronomical inquiry is touched upon more or less; from the determination of the polar flattening of the earth to the photography of minor planets and the appearances of nebulæ. Though the author never seems at a loss to give reasons for the introduction of matter apparently not at first sight connected with the subject in hand, his reasons frequently appear to be nothing more than excuses for filling so many pages. For example, we fail to see the necessity

---

* This is misprinted in the text, but right in the table of formulæ at the end of the book.

©1894 Nature Publishing Group